2014

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5038 **D** **Your Roll No................**

Unique Paper Code : 235266

Name of the Course : **B.Sc. (Hons.) Computer Science I / B.Sc. Mathematical Sciences I / B.Sc. Physical Sciences I**

Name of the Paper : Calculus and Geometry (MAPT-202)

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the questions are compulsory.
3. Attempt any **two** parts from each question.
4. Marks of each part are indicated.

1. (a) Define Rolle's theorem and also give its geometrical interpretation. (6)

(b) Discuss the continuity of f(x) defined as (6)

$$f(x) = \begin{cases} \frac{1}{2} - x & ; \quad 0 \le x < \frac{1}{2} \\ 1 & ; \quad x = \frac{1}{2} \\ \frac{3}{2} - x & ; \quad \frac{1}{2} < x < 1 \end{cases}$$

(c) Find $\lim_{x \to 0} \frac{\left(e^{\frac{1}{x}} - e^{\frac{-1}{x}}\right)}{\left(e^{\frac{1}{x}} + e^{\frac{-1}{x}}\right)}$ as x tends to zero. (6)

P.T.O.

2. (a) Define uniform continuity of a function on the interval I. Give two examples of uniformly continuous functions. (6)

(b) Discuss the differentiability of the function

$$f(x) = \begin{cases} 2+x & ; \quad x \geq 0 \\ 2-x & ; \quad x < 0 \end{cases}$$ (6)

(c) Find all the asymptotes of the curve

$$y^3 - x^2y - 2xy^2 + 2x^3 - 7xy + 3y^2 + 2x^2 + 2x + 2y + 1 = 0$$ (6)

3. (a) (i) Find the inflexion points, if any, of $f(x) = x^4$.

(ii) Show that $y = e^x$ is everywhere concave upward and the curve $y = \log x$ is everywhere concave downward. (7)

(b) Find the position and nature of multiple points on the curve

$$x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0.$$ (7)

(c) Trace the curves

(i) $r = a(1 - \cos\theta)$

(ii) $r = a\cos 2\theta$ (7)

4. (a) Trace the curve $y^2(a + x) = x^2(3a - x)$. (6)

(b) If $I_n = \int e^{-x}x^n\,dx$, show that $I_n = -e^{-x}x^n + nI_{n-1}$.

Hence show that $\int_0^\infty e^{-x} x^n \, dx = n!$, n being any positive integer. (6)

(c) Find the area included between cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ and it's base. (6)

5. (a) Show that length of loop of the curve $3ay^2 = x(x-a)^2$ is $\frac{4a}{\sqrt{3}}$. (6)

(b) Describe the graph of the equation

$16x^2 + 9y^2 - 64x - 54y + 1 = 0$. (6)

(c) Find equations of the hyperbolas with asymptotes

$y = \pm 3/2$; $b = 4$. (6)

6. (a) Rotate the coordinate axes to remove xy-term in the equation

$$x^2 + 2\sqrt{3}\,xy + 3y^2 + 2\sqrt{3}\,x - 2y = 0$$

and sketch its graph. (6½)

(b) (i) Find the equation of the sphere that has radius 3 and is tangent to the three coordinate planes.

(ii) Prove that div (kF) = k div F. where k is a constant and $F = F(x, y, z)$. (3+3½)

(c) (i) Sketch the graph of $4z^2 = x^2 + 4y^2$, name it and show the traces of the graph in the planes $z = \pm 1$.

(ii) For $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ prove that

$$\nabla(1/\|\mathbf{r}\|) = -\mathbf{r}/\|\mathbf{r}\|^3$$

(3+3½)